

MATHS
NO PROBLEM!



at



*'Teaching for mastery in primary
mathematics'*

With acknowledgements to Dr Ban Har Yeap.

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KEY:

General text

Research

Key T dialogue

WHAT IS MATHS NO PROBLEM?

5 abilities children need to acquire (the muscles for doing maths):

1. **Communication** = 'Explain it!'
2. **Visualisation** = The mind's ability to see – "Can you imagine?"
3. **Making connections** and eventually, **generalising**
4. **Number sense** = Knowing how to break numbers down
5. **Meta-cognition** = Can chn manage their own learning?

The 3 main problems identified with maths:

1. **Teaching is characterised by rote memorisation.**
2. **Chn learn procedures without knowing the meaning behind them** (conceptual vs relational understanding).
3. **Maths is confused with arithmetic** – the belief that you are extending chn by giving them bigger numbers.

...With this current way of teaching, chn may appear secure, but this is artificial. **Research shows that we lose 30% of chn by secondary** (they show gaps and begin to struggle).

Effective maths classrooms provide time for:

1. **Exploration** (Very accessible! Easy to anchor, difficult to leave.)
2. **Structuring**
3. **Journaling**
4. **Reflection**
5. **Practice**

The MNP textbooks target these 5 areas...

Why should we use the MNP textbooks?

- The lessons are carefully structured = follow a **spiral approach**.
 - They move chn forward with ideas in a carefully scaffolded way so that chn do not fall through the gaps.
- They can be used as a **reader**.
 - Reading = making connections to the physical world (aids visualisation).
 - Chn learn to explain their reasoning by reading.
 - Many chn do not make sense of symbols as most of the time it has been spelt to them. **The T needs to read not spell**.
Eg. Add a story to $51 \div 3$ - We don't say, "51 divided by 3?" Instead we say, "51 ice creams shared equally into 3 boxes is the same as to say, how many do we have in each box?"
- They can be used as a tool for Ts who are less confident in their subject knowledge.
Ts need to be able to pre-empt misconceptions and be confident enough to question chn's responses. Chn will suffer if their T has low SK. **Remember: Misconceptions are learnt!**

What is the theory underpinning MNP?

- 📖 *Dienes* - Chn learn through exploration.
- 📖 *Vygotsky* - Chn learn through social interaction.
- 📖 *Piaget* - Chn learn when immersed in an idea (focus on a single problem) for a long period, as chn can make connections.
- 📖 *Bruner* - CPA approach (concrete experience, pictorial representation, abstraction).
 - A spiral approach allows chn a better chance to grasp concepts well. Each day, the books introduce something new but the central concept stays the same.
- 📖 *Skemp* - Constructing meaning so that the skill will happen (relational understanding).
- 📖 *Baroody* - Research shows chn **remember** through 3 stages:
 1. Counting / modelling (concrete approach).
 2. 'Figuring out' = mental exercise to build cognitive capacity to remember - deriving answers using reasoning based on known facts.
 3. Remember / mastery = efficient production of answers.

Ultimate GOAL of MNP

= Average learners perform at a very high level, and weaker learners perform at an average level. We will have very few / no struggling learners because we will take care when unfolding concepts.

(This will not happen overnight; we are at the beginning of a long journey.)

= Focus on a single problem but look at it from multiple perspectives.

EXPLORATION = Extremely important!

- Present chn with a visual representation of the problem.
- Present chn with concrete resources to help them explore the problem.
- Ask chn to work with their partner (mixed-ability) to devise different methods to attempt the problem (encourage choice / make the problem accessible to all).
 - “You don’t have to agree with your friend, but you agree to listen to them.”
 - “You may not understand your friend, but do you have one method we have discussed that you do?”

Note: Do not worry about efficiency when allowing chn to choose own methods! The textbook uses questions and numbers that encourage chn to use a particular / most efficient method. Chn will naturally move to the quickest method and won’t revert. Let the process happen!

- Offer invitations to talk. Chn negotiate for meaning – *this process works best with a peer; it rarely happens when interacting with teacher as they are an authority figure.*
- Question EVERYTHING.
 - Do not ask questions to get an answer. Ask questions to check. “Are you sure?” “Is it right?”
- The T rarely explains.
- Give chn time. Chn’s confidence comes from having space to practice before putting ideas forward on a public platform.
- Only tie their hands if it absolutely necessary! Try not to put unnecessary cognitive demands on chn – let them do what they want even if it doesn’t count towards their final learning. Let chn play!

THINK: How do you present the problem? Chn will struggle to **‘Explain it!’** if they see the answer straight away. Ask chn to generate questions and allow use of resources to make explaining second nature.

Note: In Focus can come later in the lesson (after learning) – Eg. To **‘Use it’** in another context – “Is this the problem we have been solving all along?”

= Chn to explain their solution to the problem.

STRUCTURING:

- T rarely explains, they allow the chn to.
T adds layers to structure chn's responses / returns chn's responses in a better, more polished form.

The pebble analogy (Dr Yeap Ban Har)

If the T explains everything, chn go home with new pebbles = this is a burden!

If the T adds layers to chn's explanations, chn go home with the same pebble, but polished.

Using modelled examples from the book:

- "Let's have a look at how our friends in the book solve the problem."
- "My friend was doing this, I need help understanding..." "Which method have they used?"
"Have they used one of the methods that we identified?"

JOURNALING & REFLECTION

'Maths journal opportunity' – Chn to record favourite methods which can be used as a tool for Guided Practice

Let chn record in a way that is comfortable to them because...

1. The way that they explain it is far more important!
2. Conceptually there is no difference!

Note: Ideally, 'In Focus' and 'Let's Learn' should take 30 minutes. This is where the deepening takes place. 30 minutes provides sufficient support for ALL learners.

GUIDED PRACTICE

= Practice is for consolidation and fluency, *not* learning.

PRACTICE

- Present Qs from the book – “Discuss with a friend how you might attempt these.”
(Allow 5-10 mins to solve.)
 - In practice, chn attempt similar problems but they are slightly different each time. Never give them the same questions – **Variation *not* repetition.**
 - We hardly drill in maths because it has very little use. (It is only good if the final performance is the same as the rehearsal eg. synchronised swimmers!)
- Practice is followed by a mini-plenary (this is more important than the Guided Practice).
 - Use this as an opportunity to address and critique different methods.
 - “I need your advice on a few things...” “Does the method you chose work for all calculations?” “Is there a better method to use work out one of these questions?”

Note: Questions are planted in the book to encourage chn to not use the standard algorithm. E.g. For $300 - 49$, there is a more efficient way. We do not want chn to become rigid in using one method!

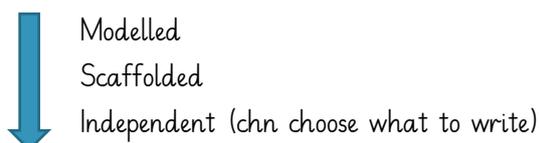
Guided Practice should take approx. 20 minutes.

INDEPENDENT APPLICATION

= Chn apply their learning independently

Maths journals are to be used to help chn practice articulating their ideas, not for simply recording their answers! It is for the final version of their explanation after their exploration and discussion.

Throughout year, chn’s journal work should be:



Note: Use questions from workbook to generate resources as they are carefully structured.

Independent Application should take approx. 5-10 minutes only.

USE OF OUR MASTER'S GLASSES



'Convince me' – Chn identify their partner's misconceptions and reason why their answer is correct but their partner's is not.

'Explain it' – Chn sit in mixed ability pairs and are regularly prompted to explain their methods to their partner. All children are provided opportunities to become a 'Tiny teacher' within a lesson, taking responsibility for their own and other's learning.

'Use it' – Chn are challenged to apply their learning, using it within another context.

'Prove it' – Chn are challenged to independently prove that they have reached the correct answer i.e. they may choose to use another method to achieve the same answer.

Chn should now actively seek alternative solutions to problems and appreciate that there are multiple solutions to any one problem.

Classifying learners

- ❖ **Struggling learners = On the runway** (making a lot of mistakes)
 - These learners need to develop their sub-skills.
 - They need a lot of T directed questions to develop their conceptual understanding.
- ❖ **Average learners = Taking off**
 - These learners take time if they have only recently learnt something – do not meddle with these learners, allow them time!
- ❖ **Advanced learners = Cruising and need some turbulence!**
 - These learners need planned opportunities for enrichment.



Categorising truly advanced learners (assessment of process not product)

1. They are always able to give you a **physical model** – e.g. add a story to / give an example of 8×3 – it is not just about knowing the number facts!
2. They are always able to give a **visual model**, e.g. a picture of a bar model.
3. They have the ability to **explain themselves orally** and can think on the spot.
4. They have the ability to **explain their thinking in a written form**.
5. They are able to **challenge themselves!**

Assessment of these learners

Key questions:

- Can they compute?
- Can they **Use it** in a familiar situation?
- Can they **Use it** in an unfamiliar situation?

Assessment for learning

Questions on the worksheets are carefully constructed to assess:

1. Can chn correctly perform the skill?
2. Do chn have the ability to take perspective / perform the skill in a certain way? For this, chn need to be able to **comprehend**.

Note: Advanced learners should never be introduced to the worksheet early as they need time to learn things meaningfully. – MISCONCEPTION IN TEACHING

HOW DO WE CATER FOR ALL LEARNERS?

Differentiate the environment

- **Struggling learners**
 - Create a safe environment where chn can make mistakes / explore the problem.
 - Create choice – always accept their method even if it is not the best (do not shut them down). At some point, the T can teach these learners to be critical of their methods.
 - Allow them processing time to gain ownership – do not meddle!
- **Advanced learners**
 - Make the environment less safe for them through questioning - “Are you sure?” “Is that true ‘for sure’, or ‘for now?’” “Is it true for more cases?” (Can chn generalise?)
 - Challenge their thinking!

Support the ‘struggling learners’

= This group will emerge within a lesson as ‘not yet secure’ - they cannot complete the final task on their own.

- Convert their Independent Application phase to Guided Practice with T.
- Chn will still attempt the same task but it will be scaffolded / directed by the T.
T will break the task down to allow chn to develop the sub-skills required for the main task.
- T to model how one responds when we do not understand – “I’m not sure that I understand that method, but...”

What does the research say?

- 📖 *Vygotsky* - Work with a friend (social interaction / role model).
- 📖 *Bruner* - Provision of concrete material during initial learning to develop conceptual understanding.
- 📖 *Bruner* - Pictorial representation during all learning – our mind processes this far better than symbolic representation.
- 📖 *Dienes* - Allow for a variety of methods – provide open ended problems.

Challenging 'advanced learners'

= Often these learners are competent at calculation, but not with number sense.

- Acceleration Vs Enrichment (stay with the **same** task).

If they finish independent application early:

- "Do you think you can invent a new method?" "Can you name it after yourself?" (One not easily accessible to others.)
- "Pick your favourite calculation and write a story for it." (One that resembles the 'In Focus' problem.) = Problem posing, making connections.
- 'Explain it!' / communicate ideas (this has an inverse correlation with ability to compute!)
"Write a note in your journal to help someone else understand what we have done today."
- Carry out research eg. "Can you get 6 different digits in one equation?"
- See *master's glasses*.

Note: If truly advanced, chn shouldn't be dependent on T challenging them. Natural steps to progression should be natural to them.

How does the textbook cater for all learners?

- The spiral approach allows those chn that do not grasp concepts quickly the chance to do so and rapid graspers the chance for enrichment.
- The anchor tasks are difficult to leave. They allow time for struggling learners and provide the opportunity for deepening for advanced learners (as there is never one answer).

Note: Toolkits should not differ in content.

POG could be:

1. Steps needed to get to year objective e.g. Year 2 to Year 3 to Year 4.
2. OR process through concrete to pictorial to abstract (differentiation by methods or representations).

If more literal / obvious, this will make it more accessible for some struggling learners.

EXAMPLE LESSON

'In Focus' part of a final lesson in a sequence on fractions (application in a word problem)
= Highlights the **reading** approach + presents a problem in parts (differentiated instruction)

Present In Focus problem... 😊 spent some of his savings on 🎁 and a part of the rest on 📚.

T: Do you think he spent all of his savings? [Ask questions about the problem first before introducing number.]

Add '1/5 of his savings on 🎁' – when parts are equal, we can name them.

T: Are you able to show how much the book costs? No, why?

Add '3/4 of the rest on 📚'

T: Talk to your friend...

T: I made a mistake. He spent 3/8 of the rest on a book. Talk to your friend.

Add 'as a result, 😊 had £60 left'

T: Would it be easy if I had £50?

T: What questions could you answer on this problem?

Suggested answers: How much does the present cost? How much does the book cost? What is his total savings?

T: How do you split £60 mentally? Record this. When done, please write a sentence to answer the question.

T: Can you swop journals with a partner? If they do not understand your explanations, you may need to make alterations to make it clearer.

T: O sorry, I made another mistake. He didn't spend 3/8, he spent 4/7.

[Please ask to see more exemplar lessons on a variety of topics.]

Idea for assessment in this lesson:

'Newman tool' – used to diagnose why chn are having difficulties with word problems. This information can then be relayed to parents.

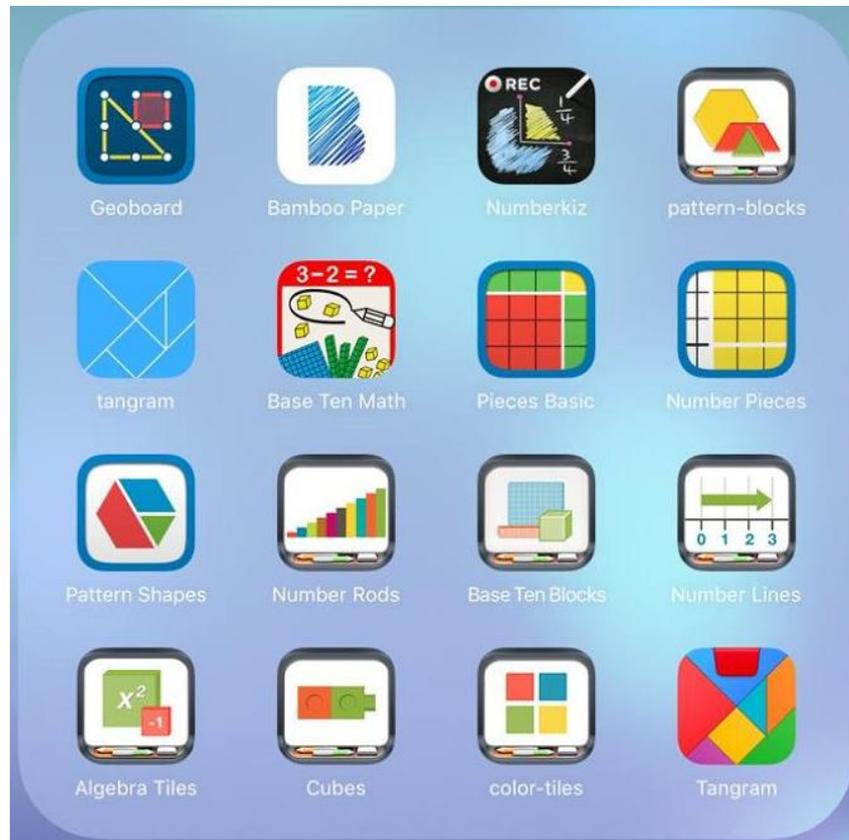
- Chn may have problems with:
 - a) Reading
 - b) Comprehending
 - c) Transformation (translating a story into its mathematical form) – chn know what they want to do but cannot transform it into a mathematical operation.
 - d) Strategies – visualisation as well as abstract computation.
 - e) Computing – final answer may be wrong but they have got to the final point.
 - f) Making sense – chn can compute (use calculation to respond to the situation) but some only get to the answer by chance. Chn need to know when to correct themselves when reflecting as their answer doesn't make sense in the given story.

Strategies for teaching word problems

- Provide context for teaching basic operations (a story).
- Provide opportunities for problem posing e.g. make a story for 3×4 .
- Provide a variety of problem situations.
- Provide problems without numbers.
- Model reading for comprehension and metacognition.
- Model the use of visual strategies including bar models.

Teaching Top Tips

- * Use the reading approach not spelling.
For $3/5$ divided by $1/5$ write -> "In 3 fifths, how many fifths are there?"
For $1/2$ divided by $1/4$ -> "In $1/2$, how many $1/4$ s are there?"
- * Chn need to realise that the number is a name e.g. write '1 half' – Chn need to see it in a literal sense first. (This has to start from Year 1!)
- * If the nouns are different, you cannot add or subtract them e.g. for $1/4 + 1/2$, nouns need to be the same.
(Misconception is $1/4 + 1/2 = 2/6$ which is like saying 1 apple + 1 orange = 2 watermelons!)
The only time we struggle with this is in multiplication as the nouns are not the same, e.g. 1 plate of 2 apples.
- * In subtraction you do not 'borrow', you 'rename' numbers so that you can work with the same nouns, eg. tens, ones.



Courtesy of Dr Ban Har Yeap